KQM Bronya Guide SPD Formula Proofs

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1 Introduction

This document offers proofs for the various SPD threshold formulas in the KQM Bronya Guide. Some definitions for terms used in the document follow.

1.1 Action Gauge, Action Value, and SPD

The concept of Action Gauge, or AG, will be referenced multiple times in this document. Action Gauge is a hidden value that represents, in essence, 'how far a character is from their next turn.' By default, all characters and enemies have their Action Gauge reset to 10,000 when their turn ends.

Action Advance and Action Delay operate directly on Action Gauge. If a character receives a 20% Action Advance, their current Action Gauge is reduced by 2,000, because 2,000 is 20% of 10,000. If a character receives a 30% Action Delay, their current Action Gauge is increased by 3,000 - and this can even go beyond 10,000.

Action Value, or AV, is the numerical value shown on the turn order. It is essentially the 'time left' before a character or enemy reaches their next turn.

A character's SPD determines how 'quickly' they consume AG. For every X AV that passes, a character's AG is reduced by $X * SPD$.

Action Gauge, Action Value, and SPD are all related by a single formula:

$$AG = AV * SPD,$$

which of course can be rearranged as needed to solve for any of the three terms:

$$AV = \frac{AG}{SPD}$$

to solve for Action Value, and

$$SPD = \frac{AG}{AV}$$

to solve for SPD.

These formulas are used extensively throughout this document.

For a more thorough explanation of these concepts and details on how these concepts work in-game, see the KQM SPD Guide.

1.2 Definitions

Let $Talent\%$ be the Action Advance granted by Bronya's Talent, represented in decimal form. For example, at Level 10, Bronya's Talent grants her a 30% Action Advance upon using Basic ATK, which would be represented as 0.3.

At Level 1 (the minimum level), Bronya's Talent grants 15% Action Advance, and at Level 12 (the maximum level with Eidolons included), Bronya's Talent grants 33% Action Advance. Thus, for the purposes of the calculations in this document, we bound $Talent\%$ within the range $[0.15, 0.33]$.

Let $DamageDealerSPD$ be the total SPD of the damage dealer after any **permanent** SPD buffs but before any **temporary** SPD buffs. In this document, there is only one temporary SPD buff considered for the damage dealer, which comes from Bronya's second Eidolon. Accounting for any other such buffs is left as an exercise to the reader.

Let $BronyaSPD$ be Bronya's total SPD after any **permanent** SPD buffs but before any **temporary** SPD buffs. In this document, no temporary SPD buffs are considered for Bronya. Accounting for any such buffs is left as an exercise to the reader.

2 Pre-E2 Bronya

2.1 Slow Skillspam

In a Slow Skillspam playstyle, one wishes for Bronya to advance the damage dealer, and then for the damage dealer's next action to come before Bronya's next action. This allows the damage dealer to take two actions per Bronya Skill, effectively doubling their action count.

In order for the damage dealer's next action to come before Bronya's next action, we must have

$$DamageDealerTotalAV < BronyaTotalAV.$$

From the introduction, we know that $AV = \frac{AG}{SPD}$. We can then transform this inequality:

$$\frac{DamageDealerTotalAG}{DamageDealerSPD} < \frac{BronyaTotalAG}{BronyaSPD}.$$

We know that both the damage dealer and Bronya must each consume $10,000$ AG total across the full rotation, so we can substitute that value to get

$$\frac{10,000}{DamageDealerSPD} < \frac{10,000}{BronyaSPD}.$$

Doing a little algebraic manipulation, we see that

$$\frac{10,000 * BronyaSPD * DamageDealerSPD}{DamageDealerSPD} < \frac{10,000 * DamageDealerSPD * BronyaSPD}{BronyaSPD}.$$

Simplifying and dividing both sides by $10,000$, we have

$$BronyaSPD < DamageDealerSPD,$$

which is the formula seen in the guide.

2.2 Slow Alternating

In a Slow Alternating playstyle, we have a repeating sequence of 5 actions:

- Bronya Skill (targets Damage Dealer)
- Damage Dealer Action
- Damage Dealer Action
- Bronya Basic ATK
- Damage Dealer Action
- (repeat)

How do we tune Bronya's and the damage dealer's SPD to achieve this sequence? First, tally the Action Gauges (AG) consumed over the full rotation for both Bronya and the damage dealer. Without any Action Advance, a character consumes $10,000$ AG per turn.

After the turn that is advanced by Bronya's Skill, the damage dealer takes action two more times, after which the rotation ends. This means the damage dealer consumes $20,000$ AG over the full rotation.

Over the same period, Bronya uses one Skill and one Basic ATK. Because the rotation begins with her Skill, she must consume the full AG of both actions. Bronya's Talent grants her a certain amount of Action Advance when she uses her Basic ATK, which reduces the total amount of AG she must consume to reach her next turn. Let $Talent\%$ be defined as in section 1.2 above. Then, with one Skill and one Basic ATK, Bronya consumes

$$10,000 + 10,000 * (1 - Talent\%)$$

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$$= 10,000 * (2 - Talent\%)$$

AG over the full rotation.

Remember that we want Bronya to complete this rotation slightly slower than the damage dealer - if not, her turn would come before the damage dealer's third action, which would waste an entire action and up to 10,000 of the damage dealer's AG. Thus, like before, we want

$$DamageDealerTotalAV < BronyaTotalAV.$$

Since $AV = \frac{AG}{SPD}$, we can substitute:

$$\frac{DamageDealerTotalAG}{DamageDealerSPD} < \frac{BronyaTotalAG}{BronyaSPD}.$$

Then, substituting in values for the damage dealer's and Bronya's total AG over the rotation, we get

$$\frac{20,000}{DamageDealerSPD} < \frac{10,000 * (2 - Talent\%)}{BronyaSPD}.$$

Multiplying both sides by $BronyaSPD * DamageDealerSPD$ and simplifying, we have

$$BronyaSPD * 20,000 < DamageDealerSPD * 10,000 * (2 - Talent\%).$$

Finally, divide both sides by $20,000$ to get

$$BronyaSPD < DamageDealerSPD * (1 - \frac{Talent\%}{2}),$$

which is the formula seen in the guide.

2.3 Fast Alternating

In a Fast Alternating playstyle, we have a repeating sequence of 4 actions:

- Bronya Skill (targets Damage Dealer)
- Damage Dealer Action
- Bronya Basic ATK
- Damage Dealer Action
- (repeat)

To tune Bronya's and the damage dealer's SPD for this playstyle, we can use the same idea we used for the Slow Alternating playstyle. This time, the damage dealer is only taking one turn after the turn advanced by Bronya's Skill

before the rotation ends, which means they only consume $10,000$ AG over the full rotation.

Bronya, like in a Slow Alternating playstyle, uses one Skill and one Basic ATK. Since her action sequence is exactly the same in this rotation as in the Slow Alternating rotation, the formula is the same: Bronya consumes

$$10,000 * (2 - Talent\%)$$

AG over the full rotation.

In order to ensure that Bronya completes this rotation slower than the damage dealer, we again want

$$DamageDealerTotalAV < BronyaTotalAV.$$

Since $AV = \frac{AG}{SPD}$, we can substitute:

$$\frac{DamageDealerTotalAG}{DamageDealerSPD} < \frac{BronyaTotalAG}{BronyaSPD}.$$

Once again, substituting the damage dealer's and Bronya's total AG (note that the damage dealer's total AG is only $10,000$ this time):

$$\frac{10,000}{DamageDealerSPD} < \frac{10,000 * (2 - Talent\%)}{BronyaSPD}.$$

Using the same trick as before, we have

$$BronyaSPD * 10,000 < DamageDealerSPD * 10,000 * (2 - Talent\%).$$

And dividing both sides by $10,000$ yields

$$BronyaSPD < DamageDealerSPD * (2 - Talent\%),$$

which is the formula seen in the guide.

2.4   Fast Double Alternating

In a Fast Double Alternating playstyle, we have a repeating sequence of 6 actions:

- Bronya Skill (targets Damage Dealer)
- Damage Dealer Action
- Bronya Basic ATK
- Damage Dealer Action
- Bronya Basic ATK

- Damage Dealer Action

- (repeat)

Like in the Slow Alternating playstyle, the damage dealer takes two turns after the turn advanced by Bronya's Skill before the rotation ends. Thus, the damage dealer will consume $20,000$ AG over the full rotation.

Unlike the two Alternating playstyles, however, Bronya now uses one Skill and *two* Basic ATKs. This means she will consume

$$10,000 + 2 * 10,000 * (1 - Talent\%)$$
$$= 10,000 * (3 - 2 * Talent\%)$$

AG over the full rotation.

In order to ensure, as in the other cases, that Bronya completes this rotation slower than the damage dealer, we want

$$DamageDealerTotalAV < BronyaTotalAV.$$

Substituting as usual:

$$\frac{DamageDealerTotalAG}{DamageDealerSPD} < \frac{BronyaTotalAG}{BronyaSPD}.$$

Substitute values for total AG consumed over the rotation:

$$\frac{20,000}{DamageDealerSPD} < \frac{10,000 * (3 - 2 * Talent\%)}{BronyaSPD}.$$

Do some algebra:

$$BronyaSPD * 20,000 < DamageDealerSPD * 10,000 * (3 - 2 * Talent\%).$$

Divide both sides by $20,000$ to get

$$BronyaSPD < DamageDealerSPD * (\frac{3}{2} - Talent\%),$$

which is the formula seen in the guide.

3   Post-E2 Bronya

Bronya's second Eidolon grants the character she advances with her Skill a 30% SPD increase until their next action ends. Like any other percentage-based SPD buff, this SPD increase scales with their base SPD, not their total SPD. We will define a variable representing the SPD of the damage dealer during the period they have the SPD buff from Bronya's E2.

Let $BuffedSPD$ be defined as

$$DamageDealerSPD + 0.3 * DamageDealerBaseSPD.$$

3.1 E2 Slow Alternating

Working with a 1-turn SPD buff in a multiple-turn rotation can be somewhat tricky. The damage dealer is still consuming $20,000$ AG over the full rotation, but their SPD differs across the two turns. This means we have to do some math when converting from AG to AV.

Bronya's SPD remains constant throughout the whole rotation, so as in the Slow Alternating rotation without E2, she consumes $10,000*(2-Talent\%)$ AG over the full rotation.

Now, to derive Bronya's relative SPD value. As usual, we want

$$DamageDealerTotalAV < BronyaTotalAV.$$

But since the damage dealer's SPD is not constant, we cannot simply divide their total AG by their SPD value to get their total AV. Instead, we have to calculate their AV for each of the two turns separately, and then add the two together:

$$DamageDealerTotalAV = \frac{Turn1TotalAG}{BuffedSPD} + \frac{Turn2TotalAG}{DamageDealerSPD}.$$

We know that each of the damage dealer's turns consumes $10,000$ AG, so we can substitute:

$$DamageDealerTotalAV = \frac{10,000}{BuffedSPD} + \frac{10,000}{DamageDealerSPD}.$$

Substituting this into the previous equation and substituting Bronya's total AG, we then have

$$\frac{10,000}{BuffedSPD} + \frac{10,000}{DamageDealerSPD} < \frac{10,000*(2-Talent\%)}{BronyaSPD}.$$

Take the reciprocal of both sides, and multiply by $10,000$ (recall that we have to reverse the inequality when we do this):

$$\frac{10,000}{\frac{10,000}{BuffedSPD} + \frac{10,000}{DamageDealerSPD}} > \frac{10,000}{\frac{10,000*(2-Talent\%)}{BronyaSPD}}.$$

Let's look at just the left-hand side for a moment. That's

$$\frac{10,000}{\frac{10,000}{BuffedSPD} + \frac{10,000}{DamageDealerSPD}}.$$

Now, we have to do something a bit weird: Multiply each term in the denominator by 1, but we disguise the 1 as $\frac{BuffedSPD*DamageDealerSPD}{BuffedSPD*DamageDealerSPD}$. This gets us

$$\frac{10,000}{\frac{10,000*DamageDealerSPD}{BuffedSPD*DamageDealerSPD} + \frac{10,000*BuffedSPD}{BuffedSPD*DamageDealerSPD}}.$$

Since the two terms in the denominator now have the same denominator themselves, we can combine them into a single fraction:

$$\frac{10,000}{\frac{10,000*(BuffedSPD+DamageDealerSPD)}{BuffedSPD*DamageDealerSPD}}.$$

Now we simplify this fraction further:

$$\frac{BuffedSPD * DamageDealerSPD}{BuffedSPD + DamageDealerSPD}.$$

Remember, this was just the left-hand side of the inequality. Let's bring the right-hand side back:

$$\frac{BuffedSPD * DamageDealerSPD}{BuffedSPD + DamageDealerSPD} > \frac{10,000}{\frac{10,000*(2-Talent\%)}{BronyaSPD}}.$$

This right-hand side can also be simplified:

$$\frac{BuffedSPD * DamageDealerSPD}{BuffedSPD + DamageDealerSPD} > \frac{BronyaSPD}{2 - Talent\%}.$$

Now all we have to do to solve for Bronya's SPD is multiply both sides by $2 - Talent\%$:

$$\frac{BuffedSPD * DamageDealerSPD}{BuffedSPD + DamageDealerSPD} * (2 - Talent\%) > BronyaSPD$$

Turn the inequality around to put $BronyaSPD$ on the left-hand side, and we have

$$BronyaSPD < \frac{BuffedSPD * DamageDealerSPD}{BuffedSPD + DamageDealerSPD} * (2 - Talent\%),$$

which is the formula seen in the guide.

3.2   E2 Fast Alternating

Fortunately, the Fast Alternating playstyle is much simpler. Since the damage dealer takes only one turn after the turn advanced by Bronya's Skill before the rotation ends, they have the 30% SPD increase from Bronya's E2 the entire time. This means that the damage dealer's SPD is effectively equal to their buffed SPD at all times, so we can take the equation from the Pre-E2 Fast Alternating formula and simply substitute in the damage dealer's buffed SPD value in place of their normal SPD. Recall that the formula for a Pre-E2 Fast Alternating Bronya said that

$$BronyaSPD < DamageDealerSPD * (2 - Talent\%).$$

Thus, if Bronya is E2:

$$BronyaSPD < BuffedSPD * (2 - Talent\%),$$

which is the formula seen in the guide.

3.3 E2 Fast Double Alternating

In the same vein as the Slow Alternating playstyle, the damage dealer consumes $20,000$ AG over a full Fast Double Alternating rotation, but their SPD again differs across the two turns.

Likewise, Bronya's SPD remains unchanged throughout the rotation, so as in the Fast Double Alternating playstyle without E2, she consumes $10,000 * (3 - 2 * Talent\%)$ AG over the full rotation.

The derivation for Bronya's SPD in this case is essentially exactly the same as for the E2 Slow Alternating playstyle. The steps will be presented without commentary, and any intermediate steps are left as an exercise for the reader. As usual, we start with

$$DamageDealerTotalAV < BronyaTotalAV$$

$$\implies \frac{10,000}{BuffedSPD} + \frac{10,000}{DamageDealerSPD} < \frac{10,000 * (3 - 2 * Talent\%)}{BronyaSPD}.$$

$$\implies \frac{10,000}{\frac{10,000}{BuffedSPD} + \frac{10,000}{DamageDealerSPD}} > \frac{10,000}{\frac{10,000*(3-2*Talent\%)}{BronyaSPD}}$$

$$\implies \frac{BuffedSPD * DamageDealerSPD}{BuffedSPD + DamageDealerSPD} > \frac{BronyaSPD}{3 - 2 * Talent\%}$$

$$\implies BronyaSPD < \frac{BuffedSPD * DamageDealerSPD}{BuffedSPD + DamageDealerSPD} * (3 - 2 * Talent\%),$$

which is the formula seen in the guide.